# HOMEWORK 5 - ANSWERS TO (MOST) PROBLEMS

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## Section 3.1: Derivatives of polynomials and exponential functions

**3.1.11.** $y' = -\frac{2}{5}x^{-\frac{7}{5}}$

**3.1.13.** $V'(r) = 4\pi r^2$, which is the surface area of sphere! What a coincidence - or is it? ;)

**3.1.20.** $f'(t) = \frac{1}{2\sqrt{t}} + \frac{1}{2t\sqrt{t}} = t^{-\frac{1}{2}} + \frac{1}{2}t^{-\frac{3}{2}}$

**3.1.32.** $y' = e^{x+1}$

**3.1.35.** $y' = 4x^3 + 2e^x$, so $y'(0) = 2$, and so the equation of tangent line is $y - 2 = 2(x - 0)$, i.e. $\boxed{y = 2x + 2}$ and equation of normal line is $y - 2 = -\frac{1}{2}(x - 0)$, i.e. $\boxed{y = -\frac{1}{2}x + 2}$ (remember that the normal line still goes through $(0, 2)$, but has slope = the negative reciprocal of the slope of the tangent line)

**3.1.49.**

(a) $v(t) = s'(t) = 3t^2 - 3$; $a(t) = v'(t) = 6t$
(b) $a(2) = 12$
(c) $v(t) = 0$ if $t = 1$ or $t = -1$, but $t > 0$ (negative time doesn't make sense), so $t = 1$, and $a(1) = 2$

**3.1.54.** First of all $y' = \frac{3}{2}\sqrt{x}$ and first find a point $x$ where $y'(x) = 3$ (remember that two lines are parallel when their slopes are equal, and the slope of $y = 1+3x$ is 3). So you want $\frac{3}{2}\sqrt{x} = 3$, so $\sqrt{x} = 2$, so $x = 4$. Now all that you need to find out is the slope of the tangent line to the curve at 4. The equation is: $\boxed{y - 8 = 3(x - 4)}$ (because from the above calculation the slope is 3, and the tangent line goes through $(4, f(4)) = (4, 8)$)

**3.1.76.** Ask me about that during office hours!

## 1. Section 3.2: The product and quotient rules

**3.2.15.** $y' = \frac{2t\left(t^4-3t^2+1\right)-t^2\left(4t^3-6t\right)}{\left(t^4-3t^2+1\right)^2}$

**3.2.37.** $y' = (2r - 2)e^r + \left(r^2 - 2r\right) e^r = \left(r^2 - 2\right) e^r$

**3.2.33.** $y'(x) = 2e^x + 2xe^x$, so $y'(0) = 2$, and so the tangent line has equation: $y - 0 = 2(x - 0)$,i.e $\boxed{y = 2x}$, and the normal line has equation: $y - 0 = -\frac{1}{2}(x - 0)$, i.e. $\boxed{y = -\frac{1}{2}x}$

*Date*: Wednesday, March 2nd, 2011.

**3.2.41.** $f'(x) = \frac{2x(1+x)-x^2}{(1+x)^2} = \frac{x^2+2x}{x^2+2x+1}$, so $f''(x) = \frac{(2x+2)(x^2+2x+1)-(x^2+2x)(2x+2)}{(x^2+2x+1)^2}$, and so $f''(1) = \frac{(2+2)(1+2+1)-(1+2)(2+2)}{(1+2+1)^2} = \frac{(4)(4)-(3)(4)}{(4)(4)} = \frac{16-12}{16} = \frac{4}{16} = \boxed{= \frac{1}{4}}$

**3.2.47.**

(a) $u'(1) = f'(1)g(1) + f(1)g'(1) = (2)(1) + (2)(-1) = \boxed{0}$

(b) $v'(5) = \frac{f'(5)g(5)-f(5)g'(5)}{(g(5))^2} = \frac{\left(-\frac{1}{3}\right)(2)-(3)\left(\frac{2}{3}\right)}{4} = \frac{-\frac{2}{3}-2}{4} = -\frac{8}{12} = \boxed{-\frac{2}{3}}$

**3.2.53.** (9200)(30593) + (961400)(1400) = 1,345,960,000

## 2. Section 3.3: Derivatives of trigonometric functions

**3.3.5.** $g'(t) = 3t^2\cos(t) - t^3\sin(t)$

**3.3.13.** $y' = \frac{\cos(x)x^2 - \sin(x)(2x)}{x^4}$

**3.3.37.** We have $\sin(\theta) = \frac{x}{10}$, so $x = 10\sin(\theta)$, so $x'(\theta) = 10\cos(\theta)$, and $x'\left(\frac{\pi}{3}\right) = 10\cos\left(\frac{\pi}{3}\right) = \frac{10}{2} = \boxed{5}$

**3.3.39.** 3 (multiply the fraction by $\frac{3}{3}$ and use the fact that $\lim_{x\to 0}\frac{\sin(3x)}{3x} = 1$)

**3.3.40.** $\frac{4}{6} = \frac{2}{3}$ (multiply the numerator by $\frac{4}{4}$ and the denominator by $\frac{6}{6}$ and use the facts that $\lim_{x\to 0}\frac{\sin(4x)}{4x} = 1$ and $\lim_{x\to 0}\frac{\sin(6x)}{6x} = 1$)